Implications of Fiber Grating Dispersion for WDM Communication Systems

B. J. Eggleton, G. Lenz, N. Litchinitser, D. B. Patterson, and R. E. Slusher

Abstract— **For high bit-rate dense wavelength-division multiplexed (DWDM) applications fiber grating dispersion for the transmitted adjacent channels is shown to be detrimental and ultimately leads to a penalty. We consider design criteria for fiber grating filters in DWDM systems using both Gaussian pulses and super-Gaussian pulses that approximate square pulses that are more common in nonreturn-to-zero (NRZ) systems.**

Index Terms—**Communication system performance, disbursive channels, gratings, optical fibers, wavelength-division multiplexing.**

I. Introduction

FIBER gratings will play an important role in optical add–drop multiplexers (OADM) for wavelength-selective routing in optical networks [1] as well as "cleanup" filters to reduce crosstalk. A fiber grating operating in reflection in conjunction with a circulator provides a means of adding and/or dropping channels in a wavelength-division multiplexed (WDM) system to facilitate wavelength routing. The advantage of the fiber grating based filters is that the reflectivity of the grating can be strong over a narrow range of wavelengths and small for all other wavelengths. This results in a very low insertion loss for both the transmitted and the reflected channels. For good performance, the fiber grating must satisfy two criteria: 1) low insertion loss (the loss around the center of the stopband) and a flat-top reflection spectrum both of which can be achieved by making the grating strong, either through high refractive index changes or by increasing the length of the grating and 2) low crosstalk, i.e., the amount of residual power "leaking" from an adjacent channel must be low (typically $<$ -25 dB) which can be achieved using apodization techniques [2]. In practice, system considerations will dictate the channel spacing (currently 100 GHz is becoming the standard) and the aim is to get a flat-top channel spectrum over most of the channel bandwidth (to allow for wavelength drift of filter position or source) and have very low crosstalk. A useful parameter may be defined as the ratio of the channel 3-dB bandwidth $\Delta\nu_W$ to the channel spacing $\Delta\nu_C$:

$$x = \frac{\Delta\nu_W}{\Delta\nu_c}. \tag{1}$$

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While out of band reflections can be dramatically reduced by apodising the grating [2], the issue of grating dispersion has not been addressed. This effect is even more striking for dense WDM and high-bit rate systems where the spacing of the channels is typically of the order of $<$100 GHz. Any dispersion in such a system would introduce dispersive pulse broadening which could be detrimental to the successful operation of the fiber grating filter, ultimately resulting in a penalty.

It is the aim of this letter to analyze the dispersive properties of a fiber-grating based filter in the context of add–drop filters. We investigate both Gaussian pulses and super Gaussian pulses that are closer to a square shape and are more realistic in nonreturn-to-zero (NRZ) systems. We will show that the ratio of the channel spacing and maximum bit rate per channel is proportional to the square root of the normalized grating strength.

II. Analysis

We start our analysis by considering the dispersive properties of uniform periodic structure. Because the fiber grating is apodized we can ignore the sidelobes in the reflection spectrum and oscillations in the dispersion [3]. Dispersion in a fiber grating can be explained in terms of the Kramers–Kronig relation: The photonic bandgap is a spectral filter and has an associated dispersion relation. This photonic bandgap (or grating stopband) corresponds to detuned frequencies $-\kappa < \delta < \kappa$ where the reflection is high. Here, $\delta = (n/c)(\omega - \omega_{\mathrm{B}})$ is the detuning parameter, n is the average refractive index, c is the speed of light, ω is the frequency of operation, and ω_{B} is the Bragg frequency. The parameter κ characterizes the strength of the grating and is related to the refractive index modulation ($\kappa = \pi\Delta n\eta/\lambda_{\mathrm{B}}$), where Δn is the index modulation, η is the percent of the energy of the mode in the core, and $\lambda_{\mathrm{B}} = 2\pi c/\omega_{\mathrm{B}}$ is the Bragg wavelength. This grating stopband corresponds to the reflected channel in the filter. The dispersion of the structure is strongest near the band edge hence for channels sufficiently close to the edge, the dispersive effects cannot be neglected. For the parameters considered in this letter, higher order dispersion is negligible [3] and we only consider the quadratic dispersion given by [4]

$$\beta_2(\delta) = \left(\frac{n}{c}\right)^2 \frac{1}{\delta} \frac{\left(\frac{\kappa}{\delta}\right)^2}{\left[1 - \left(\frac{\kappa}{\delta}\right)^2\right]^{3/2}}. \tag{2}$$

Since the channel bandwidth is given by 2κ and we are interested in a detuning of one channel spacing away (see

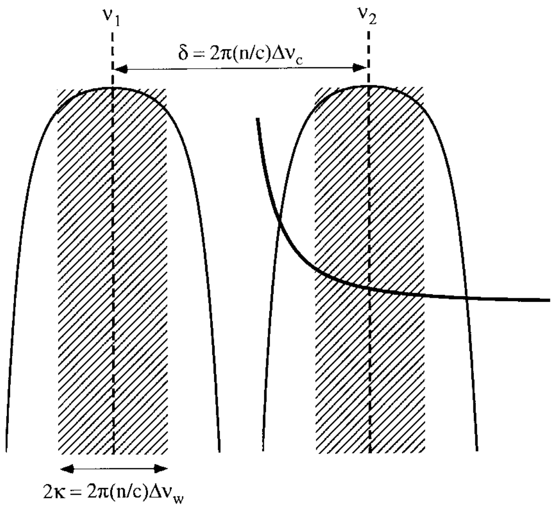

Fig. 1. A schematic of two adjacent channels in a WDM system and their relation to the fiber grating parameters. The hatched area signifies the extent of the stopband and the heavy line shows the dispersion associated with channel ν_1.

Fig. 1), we can write the ratio of κ/δ in terms of the parameter x defined earlier: $\kappa/\delta = x/2$, so that β_2 may be rewritten as follows:

$$\beta_2(\delta) = \left(\frac{n}{c}\right)^2 \frac{1}{\delta} f(x) \tag{3}$$

with

$$f(x) = \frac{\left(\frac{x}{2}\right)^2}{\left[1 - \left(\frac{x}{2}\right)^2\right]^{3/2}} \tag{4}$$

where $f(x)$ will be determined by system considerations and will show up as a numerical constant. We will first consider Gaussian pulses with field $U(t) = \exp[-1/2(t/\tau_0)^2]$ with τ_0 the half-width at the $1/e$-intensity point (τ_0 is related to the full-width at half-maximum (FWHM) intensity through $\tau_{\text{FWHM}} = 2\sqrt{\ln 2}\tau_0$). We use the critical pulsewidth τ_c [6] to characterize the effects of the dispersion on a short pulse. A transform-limited Gaussian input pulse with $\tau_0 = \tau_c$ will broaden by a factor of $\sqrt{2}$ after propagating through a medium with dispersion β_2 (ignoring higher order dispersion) and length L, and is given by

$$\tau_c = \sqrt{\beta_2(\delta)L} \tag{5}$$

where L is now the length of the grating. Next, we define a normalized grating strength parameter $\alpha = \kappa L = x\delta L/2$. We now substitute for L and for β_2 and get the following:

$$\tau_c = \frac{n}{c}\frac{1}{\delta}\sqrt{\left(\frac{2\alpha}{x}\right)f(x)}. \tag{6}$$

Finally, we relate the detuning parameter to the channel spacing $\delta = (2\pi n/c)\Delta\nu_c$ and we get our final result:

$$\tau_c = \frac{1}{\Delta\nu_c}\sqrt{\alpha}\sqrt{\frac{1}{2\pi^2}\frac{f(x)}{x}}. \tag{7}$$

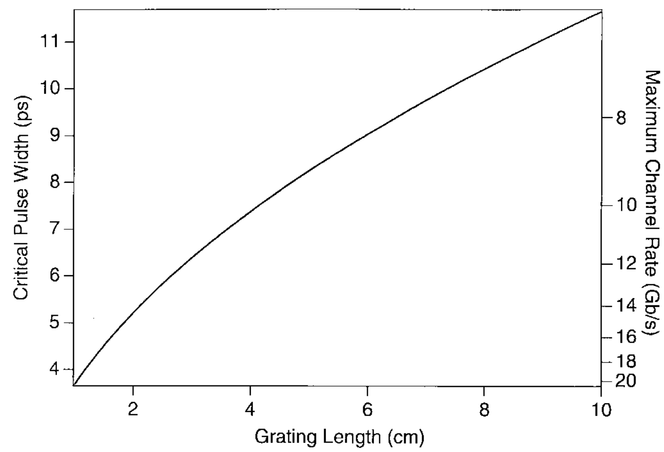

Fig. 2. The critical pulsewidth (defined in the text) as a function of the grating length. The right vertical axis gives the corresponding maximum bit rate per channel. The duty cycle $q = 25\%$ and the channel width to channel spacing ratio is $x = 75\%$.

Given the channel spacing and the x parameter we are left with only one degree of freedom, namely the grating strength α. [This can also be written in terms of the grating length as $L = \Delta\nu_c\tau_c^2\{2\pi/f(x)\}(c/n)$.]

For a transform-limited Gaussian input pulse the quadratic dispersion-induced pulse broadening factor σ is related to τ_0 through

$$\sigma = \sqrt{1 + \left(\frac{\tau_c}{\tau_0}\right)^4}. \tag{8}$$

From this relation, we see that τ_c should be made as small as possible and also that for $\tau_0 > 2\tau_c$ the broadening is negligible (less than 3%). To get a small τ_c we should try and reduce α, however, as pointed out earlier to get a flat-top spectrum and low crosstalk we need to increase α. The optimal choice for this parameter will be dictated by the system requirements. The bit rate B can be related to τ_0 through $B \approx q/2\tau_0$ where q is the duty cycle. If we want to keep the initial pulsewidth no less than twice the critical pulsewidth we get an upper limit for the allowed bit rate $B \approx q/4\tau_c$.

It should be remembered that the initial pulse spectral bandwidth has to be smaller than the channel bandwidth (this is obviously a limit that supersedes all other considerations outlined above). For a transform-limited Gaussian pulse this can be written as

$$B < \frac{1}{\pi\tau_0} < \Delta\nu_W = 2\kappa\left(\frac{c}{2\pi n}\right). \tag{9}$$

In summary, the channel spacing implies a detuning relative to the edge of the gap; closer to this edge the dispersion is higher. For a given channel spacing the band edge is specified by the x parameter and through $f(x)$ influences the dispersion $-f(x)$ is a monotonically increasing function of x and diverges as the band edge is approached. Higher dispersion leads to longer critical pulsewidths and longer input pulses, which in turn requires lower bit rates. Even though the critical pulsewidth may be reduced by using a shorter grating, this leads to smaller normalized grating strength and degradation in crosstalk and flat-top characteristics.

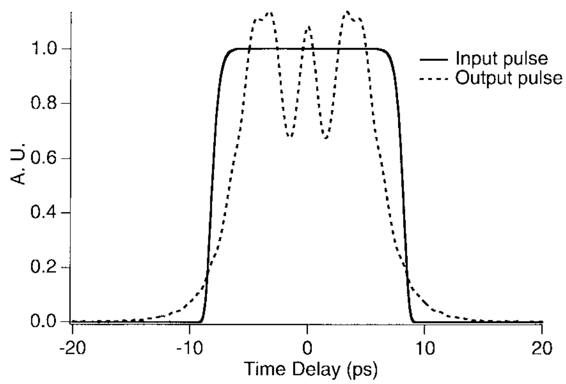

Fig. 3. A transform-limited super-Gaussian input pulse ($m = 10$; $\tau_0 = 8$ ps) and the same pulse after propagating in a 2-cm-long homogeneous dispersive medium with $\beta_2 = 2$ ps^2/cm.

We consider two numerical examples and estimate the upper limit on the bit rate per channel. As a first example we discuss the dispersion-induced broadening of a 100-GHz spaced channel with a typical x parameter of 75% yielding $\kappa = 11.4$ cm^{-1}. We assume that the input pulse is a transform-limited Gaussian pulse propagating through an apodised unchirped fiber Bragg grating. Fig. 2 shows the critical pulsewidth as given by (7) and the maximum bit rate per channel (with a duty cycle of 25%) as a function of grating length. So, for example, at a bit rate of 10 Gb/s per channel, a 4-cm-long grating will broaden the input pulse by a factor of $\sqrt{2}$. This corresponds to a κL of ~45.6, which satisfies the flat-top criterion. Note that in this case the channel bandwidth is 75 GHz and the initial pulsewidth (FWHM) is 25 ps, which corresponds to a spectral bandwidth of ~17.6 GHz (FWHM).

As a second example we consider the high aggregate bit rate systems (1 Tb/s), which were demonstrated recently [5]. There, 50 channels at 20 Gb/s per channel were used (polarization multiplexing was used to reduce the number of wavelength channels). In this example, we will consider a dense system with 25 separate wavelength channels with 50-GHz spacing. We will assume 20-ps pulses (FWHM), which at 20 Gb/s, implies a duty cycle of about $q = 40\%$. For this example, we will use an x parameter of 80%. In this case, a maximum grating length is dictated and is approximately 1.1 cm with $\alpha = \kappa L \approx 6.8$. This may be marginal as far as crosstalk and flat-top requirements.

Finally, we would like to consider the case of non-Gaussian pulses, in particular super-Gaussian pulses with the input pulse field given by [6]

$$U(t) = \exp\left[-\frac{1}{2}\left(\frac{t}{\tau_0}\right)^{2m}\right] \tag{10}$$

where m is a parameter with $m = 1$ being the Gaussian pulse considered earlier. For higher m values the pulse becomes square and, therefore, approximates well NRZ type systems where the pulses are closer to square than to a Gaussian shape. The same analysis applies in this case, but the critical pulsewidth has to be redefined to include the m dependence as follows [6]:

$$\tau_c = \left[m^2 \frac{\Gamma\left(2 - \frac{1}{2m}\right)}{\Gamma\left(\frac{3}{2m}\right)}\right]^{1/4} \sqrt{\beta_2(\delta)L}. \tag{11}$$

Here, $\Gamma(x)$ is the Gamma function and the m dependent prefactor is 1 for $m = 1$ (the Gaussian pulse case). Since super-Gaussian pulses become distorted under the influence of quadratic dispersion, the broadening, as given by (8), refers now to the root-mean-square (rms) width ($\sqrt{\overline{T^2} - \overline{T}^2}$). The prefactor increases by a factor of 2 when going from $m = 1$ (the Gaussian case) to $m = 10$ (highly super-Gaussian). Since the critical pulsewidth grows with the "squareness" of the pulse, we conclude that the constraints put on the grating will be tighter for square pulses as used in NRZ type systems. In the case of these super-Gaussian pulses, pulse broadening is not the only issue—dispersion will also cause pulse shaping even in the absence of nonlinear effects. Fig. 3 shows a transform-limited super-Gaussian input pulse ($m = 10$, $\tau_0 = 8$ ps) and the same pulse after propagating through a 2-cm-long medium with β_2 of 2 ps^2/cm. In this case, $\tau_c = 4$ ps, so the broadening is negligible, however the pulse develops significant structure, which in conjunction with fiber nonlinearities, could be very detrimental.

III. Conclusion

We have shown that in DWDM systems with many closely spaced channels, fiber grating filters may degrade pulse transmission because of the inherent dispersive character of these devices. The grating length must be designed carefully so that on the one hand it enhances the filtering properties (large κL) and on the other it minimizes dispersion induced pulse broadening (small κL). We have considered both Gaussian and super-Gaussian pulses (approximately square pulses that are more realistic in NRZ systems) and have shown that the constraints on the grating length become tighter when the pulse becomes squarer.

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