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Solitary Waves in a Nonlinear Oppositely Directed Coupler

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Abstract—The propagation of pulses in the system of two tunnel-coupled optical waveguides from optically nonlinear materials one of which has a negative refractive index, while the other one, positive, is investigated theoretically. The propagation of nonlinear waves in this structure is studied based on the model of coupled modes. For linear waves, this pair of coupled waveguides behaves as a mirror resulting in the change of direction of the energy flow upon penetration of radiation from one waveguide to the other. The solutions to the system of nonlinear equations describing the stationary propagation of the solitary wave, the gap soliton, in a particular direction are found. This soliton is formed by the coupled pair of wave packets each localized in the corresponding waveguide.

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INTRODUCTION

Two closely located waveguides can be coupled due to the tunnel penetration of light from one waveguide to the other [1, 2]. This device manufactured from materials with positive refractive index preserves the direction of light propagation and, possibly for this reason, is termed the "directed coupler."

Recently, new materials have appeared with unusual optical properties. Some of them possess negative refraction, for which both the incident and the refracted beams are at one side of the normal to the interface between two media. For Snell's law to be valid, it is necessary to assume that the refractive index of one of the media is negative. Negative refraction occurs in media in which the wave vector of the electromagnetic wave is antiparallel to the Poynting vector [3–10].

Media for which the real parts of the dielectric permittivity and magnetic permeability are simultaneously negative in some frequency range have this property of negative refraction. The existence of these media was experimentally demonstrated first in the microwave [11–14] and then in the optical [15–18] ranges. The dielectric permittivities and magnetic permeabilities of available materials with negative refractive index have nonzero real $\mathrm{Re}\,\varepsilon(\omega)$, $\mathrm{Re}\,\mu(\omega)$ and imaginary $\mathrm{Im}\,\varepsilon(\omega)$, $\mathrm{Im}\,\mu(\omega)$ parts. At present, the technological level is insufficient for creating materials with $\mathrm{Im}\,\varepsilon(\omega) \ll \mathrm{Re}\,\varepsilon(\omega)$ and $\mathrm{Im}\,\mu(\omega) \ll \mathrm{Re}\,\mu(\omega)$. Losses in available materials are sufficiently high. However, hard work aimed at decreasing losses resulted in the last year in a considerable improvement in the material quality [19]. This leaves hope for obtaining transparent materials with negative refraction in the future.

It is also known that, in materials with sufficiently strong spatial dispersion of dielectric media, waves can exist with the wave vector directed opposite to the Poynting vector [20, 21]. A consequence is a negative refraction of these waves (polaritons).

If one of the waveguides of the coupler is manufactured from a material with a negative refractive index, this device acts as a mirror [22]; the radiation that entered one waveguide leaves the device via the other waveguide. Therefore, this device can be called the oppositely directed coupler. Investigations into wave interactions in cases when such a coupler is manufactured from optically nonlinear materials may turn out to be quite fruitful. It has been shown recently [23] that the nonlinear oppositely directed coupler possesses the bistable properties related to the nonunique dependence of the transmittance (reflectivity) on the power of input radiation.

The propagation of nonlinear solitary waves in extended directed couplers has been considered and studied in detail [24–35]. In particular, it was shown [27–35] that pairs of coupled stationary solitary waves, which are sometimes termed solitons, can propagate in this waveguide structure. In reality, the equations used in these works are not completely integrable, and, for this reason, stationary wave packets are not solitons in a strict sense. Here, this term is used as a synonym of the stationary solitary wave. Since the oppositely directed coupler effectively acts as a distributed mirror, it can be expected that stationary solitary waves in the nonlinear oppositely directed coupler are similar to gap solitons [35–39].

In this work, the propagation of coupled waves in the system of two tunnel-coupled waveguides, one with negative, and the other with positive refractive index, is considered. Particular solutions to the system of coupled equations describing the evolution of these waves are found. These solutions are analogues of known gap solitons; although, unlike original gap solitons, there is no Bragg lattice here. The occurrence of a gap in the spectrum of linear waves here is due to the negative refraction of one of the waveguides.

FORMULATION OF THE MODEL

The electric field of an optical wave propagating in the system of tunnel-coupled waveguides along their common axis z is represented by the superposition of waves in each waveguide [2],

$$E(x, y, z; t) = \sum_{J=1,2}\sum_{m} A_m^{(J)}(z, t)\Psi_m^{(J)}(x, y) \times \exp[-i\omega_0 t + i\beta_m^{(J)} z]. \quad (1)$$

Here, the functions $\Psi_m^{(J)}(x, y)$ describe the transverse distribution of the mth mode of the field in the waveguide with the number $J = 1, 2$; and $A_m^{(J)}$ is the slowly varying envelope of the electric field of the corresponding mode. The parameters $\beta_m^{(J)}$ are called the propagation constants. It is assumed that the dielectric permittivity $\varepsilon^{(1,2)}(\omega_0)$ and the magnetic permeability $\mu^{(1,2)}(\omega_0)$ of the first and the second waveguides at the frequency ω_0 of the carrier wave are real. If the waveguides are manufactured from a dielectric with a nonlinear third-order susceptibility, the system of equations of coupled modes for the normalized envelopes $q_J(\zeta, \tau) = A_m^{(J)}(z, t)A_0^{-1}$ has the following form:

$$i(\hat{k}_1\partial q_1/\partial z + (1/\nu_{g1})\partial q_1/\partial t) + K_{12}q_2\exp\{+i\Delta\beta z\} + 2\pi\omega_0/c\sqrt{\mu_1(\omega_0)/\varepsilon_1(\omega_0)}A_0^2\chi_{K,\mathrm{eff}}^{(1)}|q_1|^2 q_1 = 0, \quad (2.1)$$

$$i(\hat{k}_2\partial q_2/\partial z + (1/\nu_{g2})\partial q_2/\partial t) + K_{21}q_1\exp\{-i\Delta\beta z\} + 2\pi\omega_0/c\sqrt{\mu_2(\omega_0)/\varepsilon_2(\omega_0)}A_0^2\chi_{K,\mathrm{eff}}^{(2)}|q_2|^2 q_2 = 0. \quad (2.2)$$

Here, ν_{gJ} are the absolute values of the group velocities for the Jth waveguide, and the parameters $\hat{k}_{1,2}$ are the signs of the projections of the vectors of the group velocities onto the direction of the Z axis. It is assumed that the second-order dispersion of the group velocities is insignificant. The coefficients $\chi_{K,\mathrm{eff}}^{(1,2)}$ are the nonlinear third-order susceptibilities of the first and the second channels. The coefficients K_{12} and K_{21} are the coupling constants between the modes of the adjacent waveguides, the violation of the exact phase-matching condition (the difference in the phase velocities in different waveguides) is taken into account by the parameter $\Delta\beta = \beta_m^{(2)} - \beta_m^{(1)}$. In Eqs. (2), it should be assumed that $\hat{k}_1 = +1$ and $\hat{k}_2 = -1$, if the waveguide with $J = 1$ is characterized by a positive refractive index, and the waveguide with $J = 2$ possesses a negative refraction. It is also assumed that the carrier frequency is chosen such that the following relations are satisfied: $\mathrm{Im}\varepsilon^{(1,2)}(\omega_0) \ll \mathrm{Re}\varepsilon^{(1,2)}(\omega_0)$ and $\mathrm{Im}\mu^{(1,2)}(\omega_0) \ll \mathrm{Re}\mu^{(1,2)}(\omega_0)$.

It is convenient to introduce the following normalized variables:

$$Q_1 = \sqrt{K_{21}}q_1\exp(-i\Delta\beta z/2), \quad Q_2 = \sqrt{K_{12}}q_2\exp(+i\Delta\beta z/2), \quad (3.1)$$

$$\zeta = z/L_c, \quad \tau = t_0^{-1}(t - z/V_0), \quad L_c = (K_{12}K_{21})^{-1/2}, \quad (3.2)$$

$$t_0 = L_c(\nu_{g1} + \nu_{g2})/2\nu_{g1}\nu_{g2}, \quad V_0^{-1} = (\nu_{g2} - \nu_{g1})/2\nu_{g1}\nu_{g2}. \quad (3.3)$$

In terms of the normalized variables, system of equations (2) is written as

$$i(\partial/\partial\zeta + \partial/\partial\tau)Q_1 - \delta Q_1 + Q_2 + r_1|Q_1|^2 Q_1 = 0, \quad (4.1)$$

$$i(\partial/\partial\zeta - \partial/\partial\tau)Q_2 + \delta Q_2 - Q_1 - r_2|Q_2|^2 Q_2 = 0, \quad (4.2)$$

where $\delta = \Delta\beta L_c/2$. The nonlinearity parameters are determined by the formulas

$$r_1 = \frac{2\pi\omega_0}{cK_{21}\sqrt{K_{12}K_{21}}}\sqrt{\mu_1(\omega_0)/\varepsilon_1(\omega_0)}A_0^2\chi_{K,\mathrm{ef}}^{(1)},$$

$$r_2 = \frac{2\pi\omega_0}{cK_{21}\sqrt{K_{12}K_{21}}}\sqrt{\mu_2(\omega_0)/\varepsilon_2(\omega_0)}A_0^2\chi_{K,\mathrm{ef}}^{(2)}.$$

Using the real variables $Q_{1,2} = a_{1,2}\exp(i\varphi_{1,2})$, we can pass from system of equations (4) to the equations in terms of real variables,

$$\begin{aligned}(\partial/\partial\zeta + \partial/\partial\tau)a_1 &= a_2\sin\Phi,\\ (\partial/\partial\zeta - \partial/\partial\tau)a_2 &= a_2\sin\Phi,\\ (\partial/\partial\zeta + \partial/\partial\tau)\varphi_1 &= -\delta + a_2/a_1\cos\Phi + r_1a_1^2,\\ (\partial/\partial\zeta - \partial/\partial\tau)\varphi_2 &= \delta - a_1/a_2\cos\Phi - r_2a_2^2,\end{aligned} \quad (5)$$

where $\Phi = \varphi_1 - \varphi_2$. The following conservation law follows from the amplitude equations:

$$\partial/\partial\zeta(a_2^2 - a_1^2) = \partial/\partial\tau(a_1^2 + a_2^2).$$

Therefore,

$$\partial/\partial\zeta \int_{-\infty}^{+\infty} (a_2^2 - a_1^2) d\tau = (a_1^2 + a_2^2)\Big|_{-\infty}^{+\infty}.$$

If the boundary conditions correspond to the solitary wave with similar asymptotics for $\zeta \longrightarrow +\infty$ and $\zeta \longrightarrow +\infty$, the right-hand side of this expression vanishes. In these cases, the first integral of motion is valid,

$$I = \int_{-\infty}^{+\infty} (a_2^2 - a_1^2) d\tau.$$

In the ordinary (nonlinear) directed coupler, a similar integral of motion exists; however the signs of both terms in its integrand are positive, which reflects the similarity of the direction of the energy flow in the coupled waveguides.

THE DISPERSION RELATION

If the electric field strength in the waveguides is sufficiently small, and nonlinear effects are insignificant, we can pass from (4) to the linear system

$$i(\partial/\partial\zeta + \partial/\partial\tau)Q_1 - \delta Q_1 + Q_2 = 0, \qquad (6.1)$$

$$i(\partial/\partial\zeta - \partial/\partial\tau)Q_2 + \delta Q_2 - Q_1 = 0. \qquad (6.2)$$

This system of equations determines the behavior of linear waves in the considered device. Using the Fourier transform

$$Q_{1,2}(\zeta, \tau) = (2\pi)^{-2} \int_{-\infty}^{\infty} Y_{1,2}(\omega, k) \exp\{-i\omega\tau + ik\zeta\} d\omega dk,$$

the dispersion relation for harmonic waves can be found as

$$(\omega - \delta)^2 - k^2 = 1$$

or

$$\omega(k) = \delta \pm \sqrt{1 + k^2}. \qquad (7)$$

This implies that a gap exists in the spectrum of linear waves whose width for $k = 0$ is $\Delta\omega = 2$. Harmonic waves with frequencies in this gap do not propagate through such a coupler and are reflected from it like from a Bragg mirror [40–42]. More precisely, the radiation entering one of the waveguides penetrates the second waveguide in which its direction of propagation changes in such a way that the energy flows in the opposite direction. If the two waveguides are manufactured from identical materials (characterized by positive or negative refraction), there is no gap in the spectrum, and the direction of wave propagation and of the energy flow is preserved.

A NONLINEAR STATIONARY WAVE—A QUASI-GAP SOLITON

Stationary waves correspond to the solutions to system (4) or (5) which depend on the coordinate and time via one variable $\xi = (\zeta + \beta\tau)/\sqrt{1 - \beta^2}$, with β being a free parameter. Let $\sqrt{1+\beta}a_1 = u_1$ and $\sqrt{1-\beta}a_2 = u_2$. In this case, system of equations (5) is

$$\partial/\partial\xi u_1 = u_2 \sin\Phi, \quad \partial/\partial\xi u_2 = u_1 \sin\Phi,$$

$$\partial/\partial\xi\varphi_1 = -\delta\sqrt{(1-\beta)/(1+\beta)} + u_2/u_1 \cos\Phi + \theta_1 u_1^2, \qquad (8)$$

$$\partial/\partial\xi\varphi_2 = +\delta\sqrt{(1+\beta)/(1-\beta)} - u_1/u_2 \cos\Phi - \theta_2 u_2^2,$$

where

$$\theta_1 = r_1/(1+\beta)\sqrt{(1-\beta)/(1+\beta)},$$

$$\theta_2 = r_2/(1-\beta)\sqrt{(1+\beta)/(1-\beta)}.$$

For the phase difference $\Phi = \varphi_1 - \varphi_2$, the following equation is satisfied:

$$\frac{\partial}{\partial\xi}\Phi = -2\delta/\sqrt{1-\beta^2} + (u_1/u_2 + u_2/u_1)\cos\Phi + \theta_1 u_1^2 + \theta_2 u_2^2. \qquad (9)$$

For simplicity, let us consider the case of the phase matching of interacting waves $\delta = 0$. The boundary conditions $a_{1,2} \longrightarrow 0$ for $\xi \longrightarrow \pm\infty$ choose from all solutions to system of equations (8), (9) those that correspond to solitary waves against the zero background. It follows from the equations for the amplitudes in (8) that $u_1^2 = u_2^2$, or $u_1 = \varepsilon u_2$, where $\varepsilon = \pm 1$. Therefore, Eqs. (8) and (9) are reduced to the system of two equations

$$\partial/\partial\xi u_1 = \varepsilon u_1 \sin\Phi, \qquad (10.1)$$

$$\partial/\partial\xi\Phi = 2\varepsilon\cos\Phi + \theta u_1^2, \qquad (10.2)$$

where $\theta = \theta_1 + \theta_2$. By multiplying the left- and right-hand sides of (10.2) by $u_1^2 \sin\Phi$ and using Eq. (10.1), we can obtain

$$u_1^2 \sin\Phi \partial\Phi/\partial\xi = 2\varepsilon\cos\Phi u_1^2 \sin\Phi + \theta u_1^4 \sin\Phi$$

or

$$-u_1^2 \partial\cos\Phi/\partial\xi - \cos\Phi 2u_1 \partial u_1/\partial\xi$$

$$= -u_1^2 \partial\cos\Phi/\partial\xi - \cos\Phi \partial u_1^2/\partial\xi = \varepsilon\theta u_1^3 \partial u_1/\partial\xi,$$

$$-\partial/\partial\xi(u_1^2 \cos\Phi) = \varepsilon\theta\partial u_1^4/4\partial\xi.$$

Thus, we obtain the second integral of motion

$$u_1^2(\cos\Phi + \varepsilon\theta u_1^2/4) = C_2. \quad (11)$$

Due to the boundary conditions, the integration constant is zero, and the nonzero solutions to system (10) satisfy the relation

$$\cos\Phi + \varepsilon\theta u_1^2/4 = 0. \quad (12)$$

By substituting (12) into (10.1), we obtain the equation

$$(du_1/d\xi)^2 = u_1^2[1-(\theta/4)^2 u_1^4].$$

By changing the variable $u_1 = w^{-1/2}$, this equation is reduced to the equation

$$(dw/d\xi)^2 = 4[w^2 - (\theta/4)^2],$$

which has the following solution: $w(\xi) = (\theta/4)\cosh 2(\xi - \xi_2)$. Thus, the solutions to the amplitude equations of system (8) are given by the expressions

$$u_1^2(\xi) = u_2^2(\xi) = 4/[\theta\cosh 2(\xi-\xi_2)]. \quad (13)$$

Taking into account this result, it follows from (8) that

$$\partial\varphi_1/\partial\xi = (\theta_1 - \theta/4)u_1^2(\xi)$$
$$= 3\theta_1 - \theta_2/[\theta\cosh 2(\xi-\xi_2)].$$

This yields

$$\varphi_1(\xi) = \varphi_1(-\infty)$$
$$-[1-4\theta_1/(\theta_1+\theta_2)]\arctan\exp[2(\xi-\xi_2)]/2. \quad (14.1)$$

The expression for the wave phase in the second waveguide is obtained analogously,

$$\varphi_2(\xi) = \varphi_2(-\infty)$$
$$+[1-4\theta_2/(\theta_1+\theta_2)]\arctan\exp[2(\xi-\xi_2)]/2. \quad (14.2)$$

The amplitudes $a_{1,2}$ are determined by the expressions

$$a_1^2(\xi) = 4/[\theta(1+\beta)\cosh 2(\xi-\xi_2)],$$
$$a_2^2(\xi) = 4/[\theta(1-\beta)\cosh 2(\xi-\xi_2)]. \quad (15)$$

The solutions (14), (15) describe the stationary solitary wave propagating in the extended nonlinear oppositely directed coupler under the condition of equality of phase velocities of the waves localized in each of the channels.

CONCLUSIONS

In this work, the propagation of a nonlinear solitary wave in coupled systems of two tunnel-coupled waveguides, one with a negative and the other one with a positive refractive index, is considered. In this case, the group velocities of wave packets propagating in separate waveguides are oppositely directed, and the phase velocities have the same value and direction. This property, the antidirected phase and group velocities in media with a negative refraction, results in the fact that part of the radiation is reflected, and part passes through the considered oppositely directed coupler. In the case of large electric field strengths, due to the nonlinear properties of the medium, the formation of a coupled state of wave packets from different waveguides is possible. In this case, the nonlinear interaction suppresses the effect of inversion of the wave propagation direction. In this work, particular solutions to the system of coupled equations were found which demonstrate the propagation of the coupled pair of solitary waves, the gap soliton. Each of the waves is concentrated in the separate waveguide and propagates in the direction common for both waves. The phase difference of coupled waves is varied inside in such a way that there is no energy exchange in general.

It should be noted that recent works by Stockman [43, 44] have sparked off intensive debates on the possibility itself of the existence of materials with a negative refractive index and zero losses. Without going into the discussion of this problem, we note that the assumptions made here (i.e., $\text{Im}\varepsilon^{(1,2)}(\omega_0) \ll \text{Re}\varepsilon^{(1,2)}(\omega_0)$ and $\text{Im}\mu^{(1,2)}(\omega_0) \ll \text{Re}\mu^{(1,2)}(\omega_0)$) admit the existence of losses.

The notion of the gap soliton is often related to the periodicity of the structure of a nonlinear medium in which the wave propagates. In order to underline this specific feature of the medium, the gap solitons are referred to as the Bragg solitons [39]. At the same time, it is known that the occurrence of a gap (the forbidden zone) in the spectrum of linear waves is not necessarily the consequence of the periodicity. The most well-known example is the polariton gap occurring as a result of resonance interactions of electromagnetic radiation with atoms (molecules). Nonlinear excitations localized in this forbidden zone give examples of gap solitons in media in which the periodic variation of the optical properties (the refractive index, the waveguide thickness, etc.) is absent. For this reason, a pulse of the self-induced transparency in a dense dielectric [45] could be called a gap soliton. It was shown in [35] that the system of two tunnel-coupled fiber waveguides with opposite signs of the second-order dispersion of group velocities admits the propagation of linear waves, in the spectrum of which there is a forbidden zone. A nonlinear wave, a soliton, in this waveguide is the example of the gap soliton in the medium whose parameters do not vary periodically. The stationary pulse of electromagnetic radiation considered in this paper is a novel example of such a gap soliton.

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