Analysis of spectral characteristics of photonic bandgap waveguides

A. K. Abeeluck, N. M. Litchinitser, C. Headley, and B. J. Eggleton
OFS Laboratories, 25 Schoolhouse Road, Somerset, NJ 08873
abeeluck@ofsoptics.com

Abstract: A numerical model based on a scalar beam propagation method is applied to study light transmission in photonic bandgap (PBG) waveguides. The similarity between a cylindrical waveguide with concentric layers of different indices and an analogous planar waveguide is demonstrated by comparing their transmission spectra that are numerically shown to have coinciding wavelengths for their respective transmission maxima and minima. Furthermore, the numerical model indicates the existence of two regimes of light propagation depending on the wavelength. Bragg scattering off the multiple high-index/low-index layers of the cladding determines the transmission spectrum for long wavelengths. As the wavelength decreases, the spectral features are found to be almost independent of the pitch of the multi-layer Bragg mirror stack. An analytical model based on an antiresonant reflecting guidance mechanism is developed to accurately predict the location of the transmission minima and maxima observed in the simulations when the wavelength of the launched light is short. Mode computations also show that the optical field is concentrated mostly in the core and the surrounding first high-index layers in the short-wavelength regime while the field extends well into the outermost layers of the Bragg structure for longer wavelengths. A simple physical model of the reflectivity at the core/high-index layer interface is used to intuitively understand some aspects of the numerical results as the transmission spectrum transitions from the short- to the long-wavelength regime.

©2002 Optical Society of America

OCIS codes: (060.0060) Fiber optics and optical communications; (230.1480) Bragg reflectors; (230.3990) Microstructure devices; (230.7370) Waveguides

References and links
1. Introduction

Recent advances in microstructured optical fibers (MOFs) indicate their potential for several applications for fiber-based optical devices and telecommunications. Typically, MOFs are fabricated with air holes running along the length of the fiber and distributed in the cladding in either a regular or a random fashion; MOF geometries include both hollow and solid cores [1-10]. The light guidance mechanism in the core allows these fibers to be divided into two categories: (1) air-silica MOFs [1-4] that rely on total internal reflection from the lower effective index of the cladding for light guidance in the high-index core, and (2) photonic bandgap (PBG) fibers [5,6] in which light propagates in the low-index core due to Bragg scattering off the higher effective index cladding. A different geometry of PBG fibers consists of concentric rings [11-14] of alternating high-index and low-index layers around a low-index core. This type of fiber is known as a Bragg fiber since coherent Bragg scattering from the concentric mirror stack guides light in the core. Light propagation in the OmniGuide fiber, which is a Bragg fiber with an air core, was recently demonstrated theoretically [14].

The relatively high index contrast between the silica core and the air holes of air-silica MOFs combined with the ability to control the size and distribution of these holes allows the modal areas to be varied over a wide range (1-600 µm^2, typically), hence allowing the enhancement or reduction of nonlinear effects [10]. PBG fibers are also attractive as they can serve as an alternative to current transmission fibers for telecommunications with the crucial potential advantages of lower absorption losses and reduced nonlinearities, and potential for dispersion engineering due to light guidance in the air core and manipulation by the periodic cladding region [5,14]. These fibers exhibit a spectral response that is approximately periodic with frequency, interpreted as higher order Bragg reflections from the periodic cladding.

The ability to predict the spectral transmission of these fibers is of paramount importance as it provides useful information such as the location of bandgaps (for PBG waveguides) and is hence useful in the design of new fibers and fiber-based devices. Both analytical and numerical approaches have previously been used to study some of the properties of MOFs such as dispersion, loss and modal attributes [7-12,14]. In this paper, we use numerical techniques based on a beam propagation method (BPM) [15,16] to study light transmission in PBG waveguides. Our theoretical work was motivated by a need to understand the transmission spectra of Bragg fibers and to investigate analogies with a simpler 1-D planar model. Indeed, we find that many of the spectral features of the cylindrical waveguide can be understood using a 1-D numerical model. Furthermore, the latter reveals two regimes of light propagation in PBG waveguides. For long wavelengths, Bragg scattering off the high effective index cladding determines the spectral response with modal fields penetrating into the outer layers of the cladding. The spectral characteristics are, however, different for short
wavelengths with marginal variation of the transmission spectrum with the pitch, and modal fields are confined mostly in the low-index core. An analytical model based on an antiresonant reflecting guidance mechanism is used to explain the location of the transmission maxima and minima in the short-wavelength regime.

2. Simulation method

A scalar BPM for computing the transmitted power in the core and a correlation method for calculating the modal properties of 1-D PBG waveguides were used [15,16]. In general, a vector BPM is needed to take into account the effect of polarization of the optical field. However, we show in Section 3.3 that for the purpose of computing the transmission spectra of some simple waveguides where the index contrast is as large as 0.4, the simulation results from the scalar and the vector BPMs do not differ significantly. Thus, computational time is saved by using a scalar BPM.

Scalar BPM requires the electromagnetic wave equation to be recast into the Helmholtz equation using the scalar approximation for the optical field. Using a slowly varying envelope approximation, the scalar field, \( \Psi(x, y, z) \), can be computed along the propagation direction \( z \) given an initial launch condition. The relative power, \( P(z) \), in the core is determined by the following integral over the core area: \( \int \int \Psi(x,y,z)\Psi^*(x,y,z) \, dx \, dy \). The normalized transmitted power, \( T(\lambda_0, L) \), defined as a function of the free-space wavelength (\( \lambda_0 \)) and the length of the waveguide (L), is given by

\[
T(\lambda_0, L) = \frac{\int \int \Psi(\lambda_0, x, y, L)\Psi^*(\lambda_0, x, y, L) \, dx \, dy}{\int \int \Psi(\lambda_0, x, y, 0)\Psi^*(\lambda_0, x, y, 0) \, dx \, dy}
\]

where the integral in the numerator is the power in the core at the output end of the waveguide at \( z = L \), and the integral in the denominator is the total input power at \( z = 0 \). The spectral response is acquired by consecutively scanning through a range of input optical fields with the same launch condition but with different wavelengths. In general, scalar BPM assumes paraxial propagation and small rates of change of index with propagation distance.

The modes of the waveguide can be computed using the correlation method that relies on propagating an arbitrary optical field launched asymmetrically to excite all its modes. By Fourier transformation of the correlation function between the input field and the propagated field (obtained using BPM) at any arbitrary \( z \), a spectrum of the modal propagation constants can be extracted, and hence the mode shapes can be computed [16,17].

3. Simplified 1-D model of a Bragg fiber

3.1 Structure of a Bragg fiber

The cross-section of a typical Bragg fiber [11-14] consists of a low-index core surrounded by concentric rings of high-index/low-index layers, as shown in Fig. 1 (a), where the core is solid silica with a diameter \( D \). The low-index layers of the Bragg mirror are assumed to have the same index as the core (\( n_{low} \)) while the high-index layers have an index \( n_{high} \). The Bragg mirror is further characterized by the thickness, \( d \), of each high-index layer, and the pitch, \( \Lambda \), which is the total thickness of one high-index layer and an adjacent low-index layer. Light propagates in the core through coherent Bragg scattering from the surrounding multi-layer stack and the higher the number, \( N \), of high-index/low-index layer pairs, the lower is the loss for a given index contrast (\( \Delta n = n_{high} - n_{low} \)). Typical parameters for the Bragg fiber in our simulations are as follows: \( d = 3.437 \, \mu m \), \( \Lambda = 5.642 \, \mu m \), \( D = 19.131 \, \mu m \), \( N = 10 \), \( n_{low} = 1.4 \) and \( n_{high} = 1.8 \). These parameters were chosen by comparing the Bragg fiber with an experimental PBG fiber studied by Bise et al. [6]. A cross-section of the PBG fiber is shown below the Bragg fiber in Fig. 1 (a), and it consisted of a solid silica core surrounded by ten hexagonal rings of circular air holes filled with a high-index material. The diameter of an
individual air hole was set equal to the thickness of each high-index layer of the Bragg fiber. The pitch of the PBG fiber (defined as the separation between the centers of adjacent air holes), the core diameter, and the number of rings were also the same as the corresponding parameters of the Bragg fiber. The chosen refractive index for the high-index layer in the Bragg fiber reflects that of the material filling the air holes of the experimental PBG fiber.

3.2 One-dimensional PBG waveguide model

The Bragg fiber is further simplified by reducing it to a 1-D planar waveguide, labeled \( W_1 \) and shown in Fig. 1 (b). The 1-D PBG waveguide is obtained by slicing the Bragg fiber through its center along the dotted line shown in Fig. 1 (a). Although the 1-D model speeds up the simulation and allows us to understand light propagation in the Bragg fiber intuitively, the mode structures and the absolute transmission losses of the planar waveguide and the cylindrical waveguide will be different.

The width of the low-index core in the 1-D PBG waveguide \( W_1 \) matched the fiber core diameter \( D \) of 19.131 \( \mu \text{m} \). The Bragg mirrors on either side of the core each had ten high-index/low-index layer pairs with the same values of \( d \) (3.437 \( \mu \text{m} \)) and \( \Lambda \) (5.642 \( \mu \text{m} \)) as the Bragg fiber. Moreover, the 1-D planar waveguide had the same index contrast (\( \Delta n = n_{\text{high}} - n_{\text{low}} = 0.4 \)) as the Bragg fiber.

![Fig. 1](image)

Fig. 1. (a) Cross-section of a Bragg fiber, where \( D \) is the core diameter, \( d \) is the thickness of each high-index layer of the Bragg mirror stack and \( \Lambda \) is the pitch. As discussed in the text, the material and physical parameters of the Bragg fiber were chosen with respect to an experimental PBG fiber, the cross-section of which is shown below the Bragg fiber. (b) Index profile of a planar waveguide (labeled \( W_1 \) ) obtained by slicing the Bragg fiber in (a) along the dotted line; a centered Gaussian beam was launched along the low-index core of diameter \( D \). Typical length \( L \) of the waveguide in our simulations was 5 cm.

The spectral response of the 1-D waveguide was calculated using a scalar BPM as a function of the free-space wavelength by launching a Gaussian beam of width 8 \( \mu \text{m} \) centered on the low-index core at \( z = 0 \). The propagation length \( L \) was chosen to be 5 cm, unless stated...
otherwise. The effect of material absorption was included in our simulations by assuming an absorption coefficient \(\alpha = 2.9 \text{ dB/cm}\) for the high-index material. This value for the absorption coefficient represents the loss due to absorption by the high-index material (at \(\lambda_0 = 694.3 \text{ nm}\)) filling the air holes of the PBG fiber discussed in Section 3.1. For simplicity, the same value for the material loss was assumed at all launch wavelengths.

### 3.3 Comparison of cylindrical and planar waveguide geometries

The validity of the simplification described in Section 3.2 was put to test by comparing the computed transmission spectra of a cylindrical waveguide and its planar analogue using a scalar BPM. The cylindrical waveguide chosen was a Bragg fiber in which all the high-index/low-index layer pairs had been stripped with the exception of a single high-index layer around the core. The corresponding planar waveguide had single high-index layers on either side of the core. These two simple waveguides are shown in Fig. 2 (a) while their computed transmitted spectra are shown in Fig. 2 (b). The simulation parameters were the same as those given in Sections 3.1 and 3.2. Material absorption was not considered in the simulations described in this sub-section.

![Diagram of waveguide geometries](image)

**Fig. 2.** (a) Structure of a simple cylindrical waveguide and its planar analogue. (b) Computed transmission spectra of the cylindrical and planar waveguides. (c) Comparison between scalar BPM and vector BPM for a cylindrical waveguide with the same parameters as the cylindrical waveguide in (a) except that \(L = 50 \mu\text{m}\) and \(D = 6 \mu\text{m}\).

Although the transmission losses depend on the specific geometry of the waveguide, the cylindrical and the planar waveguides have similar transmission spectra with respect to the location of their transmission minima and maxima. Thus, both geometries have similar separation of their bandgaps. The exact location of each transmission minimum is, however, very sensitive to the thickness of the high-index layer, as shown later in Sections 4 and 5. The
fact that the planar model can partially account for some of the salient features of the cylindrical waveguide led us to focus on the physics of the simpler planar structure in order to provide some physical intuition regarding light propagation and losses in PBG waveguides.

The validity of the scalar BPM in this simple case was tested by computing the transmission spectrum of the cylindrical waveguide using both the scalar BPM and a vector BPM. The simulation parameters were the same as those for the cylindrical waveguide described in Fig. 2 (a), except that \( L = 50 \mu m \) and \( D = 6 \mu m \). A Gaussian beam of diameter 3 \( \mu m \) and centered at \( x = y = 0 \) was launched at the input end of the waveguide. The vector BPM is much slower than the scalar BPM, as reflected in the need to choose a shorter propagation length and a smaller core diameter to ensure a smaller computational domain; it is shown later in Section 5 that the transmission spectrum is insensitive to changes in core diameter. Figure 2 (c) shows that the transmitted spectra computed using these two methods are not significantly different from each other, hence justifying the use of the scalar method.

4. Analytical model for 1-D planar PBG waveguides

In a previous paper [18], we demonstrated that the position of the transmission minima and maxima of the planar waveguide (and hence the cylindrical waveguide) shown in Fig. 2 can be calculated analytically using an Antiresonant Reflecting Optical Waveguide (ARROW) model. The high-index layer on either side of the low-index core behaves as a Fabry-Perot resonator in the ARROW model. A standing wave builds up in the high-index layer when \( k_{ex}d = \pi m \), \( m = 1, 2, \ldots \), where \( k_{ex} \) is the propagation constant. This corresponds to a resonant condition in the high-index layer so that light leaks out of the core, thus giving rise to the transmission minima. The transmission maxima result from antiresonant wavelengths that experience destructive interference within the high-index layer so that light is confined in the low-index core. The wavelengths (\( \lambda_m \)) corresponding to the transmission minima can be readily derived for \( \lambda << D \):

\[
\lambda_m = \frac{2n_{low}d}{m} \sqrt{\left( \frac{n_{high}}{n_{low}} \right)^2 - 1}, \quad m = 1, 2, \ldots \ldots \ldots (2)
\]

The wavelengths (\( \lambda_\ell \)) corresponding to the transmission maxima can also be calculated by considering the antiresonance condition within the high-index layer:

\[
\lambda_\ell = \frac{4n_{low}d}{(2\ell + 1)} \sqrt{\left( \frac{n_{high}}{n_{low}} \right)^2 - 1}, \quad \ell = 0, 1, 2, \ldots \ldots \ldots (3)
\]

The ARROW model, however, breaks down when a standing wave can no longer build up in the high-index core. This occurs at long wavelengths when

\[
\lambda > 2d \sqrt{n_{high}^2 - n_{low}^2} \quad (4)
\]

It is observed from the equations above that the spectral features depend critically on the thickness of the high-index layer and the index contrast. This is further investigated in the next section through numerical simulations.

5. Simulation results and discussion

5.1 Transmission spectra for long and short wavelengths

The calculated transmission spectra of the 1-D PBG waveguide \( W_1 \) are shown in Fig. 3 as a function of the free-space wavelength \( \lambda_0 \) as well as the free-space wavevector \( k_0 \) for four
different values of the pitch. For convenience, the spectra are divided into a long-wavelength regime [Figs. 3 (a) and (b)] and a short-wavelength regime [Figs. 3 (c) and (d)]. In k-space, the bandgaps are separated by an amount \( \Delta k_0 \sim 0.8 \mu m^{-1} \) with a more even band separation at shorter wavelengths [Fig. 3 (d)]. For longer wavelengths, Fig. 3 (a) shows that for a spread in pitch of about 0.8 \( \mu m \), the transmission spectrum narrows by about 1.8 \( \mu m \) for bandgaps centered about \( \lambda_0 \approx 10 \mu m \). This dependence on pitch is expected when light is guided along z by Bragg reflections from the two mirror stacks. However, for shorter wavelengths, Figs. 3 (c) and (d) indicate that the transmission spectrum is almost independent of the pitch for as large a change in \( \Lambda \) as 0.8 \( \mu m \). Moreover, the transmission spectrum is highly sensitive to variations as small as 0.25 \( \mu m \) in the value of \( d \) for a fixed \( \Lambda \) (see Fig. 4). This indicates a different regime where light appears to be confined mostly within the low-index core and the first high-index layers on either side. The calculated positions of the transmission minima using the ARROW model described in Section 4 are shown in Figs. 3 (c) and (d). Using Eq. (2), we find excellent agreement between the analytical ARROW model and the simulated results for short wavelengths, as indicated in Figs. 3 (c) and (d) by means of arrows along the horizontal axis. For the simulation parameters used, the ARROW model is valid for \( \lambda < 7.78 \mu m \), as calculated from Eq. (4).

![Fig. 3. Normalized transmission spectrum of the 1-D PBG waveguide W1 as a function of the free-space wavelength \( \lambda_0 \) [and the free-space wavevector \( k_0 \)] in the (a) [(b)] “long-wavelength” regime, and (c) [(d)] “short-wavelength” regime for four different values of the pitch. The thickness of the high-index layer in each Bragg mirror stack was fixed at \( d = 3.437 \mu m \). The predicted positions of the minima from the ARROW model are shown with small arrows along the horizontal axis.](image)
The critical part played by the first high-index layers on either side of the low-index core in the short-wavelength regime is further demonstrated by considering another 1-D planar waveguide (labeled W₂) with the following parameters: \(d = 3.437 \, \mu m\), \(\Lambda = 9.772 \, \mu m\), \(D = 16.107 \, \mu m\), and \(N = 5\). Unlike the relatively small variation in \(\Lambda\) shown in Fig. 3, it is noted that the pitch is almost twice as large as that of waveguide W₁. Moreover, the core diameter and the number of high-index/low-index layer pairs are also different. However, the index contrast and the thickness of the high-index layer remain the same. The index profiles of the two 1-D waveguides W₁ and W₂ are shown in Fig. 5 (a), and their computed transmission spectra are shown in Fig. 5 (b) for a wavelength range from 0.6-1 \(\mu m\). The prediction of the ARROW model is also shown in Fig. 5 (b).

Fig. 5. (a) Index profiles of 1-D PBG waveguides W₁ and W₂ with pitches 5.642 \(\mu m\) and 9.772 \(\mu m\), respectively. (b) Comparison between the transmission spectra of 1-D waveguides W₁ and W₂. The predicted positions of the minima from the ARROW model are indicated by small arrows along the horizontal axis.
Although the pitch of the Bragg mirrors in waveguide $W_2$ is nearly twice that of waveguide $W_1$, the two transmission spectra are almost identical in the wavelength range shown. In particular, we note that the transmission minima occur at approximately the same wavelengths for both waveguides. Also, the transmission spectrum is insensitive to the change in core diameter (the difference in core diameter of the two waveguides is about 3 $\mu$m). Moreover, there is again excellent agreement between the analytical ARROW model and the simulated results for both waveguides. Thus, the properties of the high-index layers sandwiching the core predominantly determine the spectral features of the 1-D PBG waveguide in the short-wavelength regime.

5.2 Mode profiles and spectra of the 1-D PBG waveguide $W_1$ for a wide wavelength range

Mode profiles calculated using the correlation method aided in further understanding the above results. Figures 6 (a)-(c) show the normalized mode amplitudes of the 1-D waveguide $W_1$ for three different exciting wavelengths ($\lambda_0 = 0.632 \mu$m, 5.07 $\mu$m and 11.3 $\mu$m) within different bandgaps while Figs. 6 (d)-(f) show the corresponding mode spectra. The latter display the relative intensity of each mode as a function of its propagation constant $\beta$ (bottom axis) as well as its effective index $n_{\text{eff}}$ (top axis) that are related by $\beta = n_{\text{eff}}k_0$. The relative power in each mode depends on the launch condition. The results shown were obtained by launching, at the input end of the waveguide, a Gaussian beam of width 8 $\mu$m and centered off-axis at $x = 4 \mu$m to excite the symmetric as well as the asymmetric modes. The length of the waveguide was set to $L = 2$ cm for this computation. The modes for each exciting wavelength are labeled as $m_1$, $m_2$,... with $m_1$ defined here as the fundamental mode and $m_2$,.. are subsequent higher-order modes. The large core diameter of the waveguide clearly causes it to be multi-moded. We note that the effective indices of the modes are less than the core index. Hence, the modes shown in Fig. 6 are leaky.

In the long-wavelength regime, the manifestation of Bragg scattering [11,12] is clearly revealed in the mode shapes of Fig. 6 (a) as both the fundamental and the higher-order modes extend symmetrically far into the outer layers of the Bragg reflectors. These spatially extended modes also account for the drop in the normalized transmitted power within the bandgap around $\lambda_0 \approx 10 \mu$m [see Figs. 3 (a) and (b)] since a more significant fraction of the light can leak out of the core into the Bragg mirrors. On the other side of the spectrum (the short-wavelength regime represented by $\lambda_0 = 0.632 \mu$m), the relative insensitivity of the transmission spectrum to variations in $\Lambda$ is corroborated by the tight confinement of the optical field within the central core with only very small oscillations in the first high-index layers outside of which these oscillations decay rapidly [see Fig. 6 (c)]. We note that the localization of the optical radiation occurs for the fundamental as well as the higher-order modes shown in Fig. 6 (c). For an intermediate exciting wavelength of 5.07 $\mu$m [see Fig. 6 (b)], the modes are observed to spread out more into the Bragg mirrors compared to the modes at the shorter wavelength of 0.632 $\mu$m but to a lesser extent compared to the longer wavelength of 11.3 $\mu$m. Thus, the modes of the waveguide gradually extend out of the core and penetrate into the outer layers of the Bragg mirrors as wavelength increases.

The computed modes in the short-wavelength regime of Fig. 6 (c) were compared to the modes of a modified waveguide in which all the layers but the central core and the adjacent first high-index layers were removed. The launch condition and the waveguide length were the same as above. The normalized mode amplitudes and the mode spectrum of this simpler waveguide are shown in Fig. 7. Both the modes and the mode spectrum resemble the corresponding modes and mode spectrum of the 1-D waveguide $W_1$ with ten high-index/low-index layers in each Bragg mirror. In particular, the modes are localized in the core with little energy distribution in the high-index cladding. However, computations of the transmission spectra [see Fig. 2 (b)] show that this waveguide is very lossy compared to the waveguide shown in Fig. 6 (c), indicating that the outer layers of the Bragg mirrors are crucial in reducing transmission loss even at shorter wavelengths where the modes are mostly confined to the core.
Fig. 6. Modes of the 1-D PBG waveguide $W_1$ at an exciting wavelength of (a) 11.3 $\mu$m, (b) 5.07 $\mu$m, and (c) 0.632 $\mu$m. The lateral index profile of the waveguide is also shown. The mode spectra corresponding to the exciting wavelength of (d) 11.3 $\mu$m, (e) 5.07 $\mu$m, and (f) 0.632 $\mu$m are shown next to each mode shape plot as a function of the modal propagation constant $\beta$ and the modal effective index $n_{\text{eff}}$. The modes are labeled as $m_1$, $m_2$, $m_3$, $m_2$, $m_3$, where $m_1$ refers to the fundamental mode and $m_2$, $m_3$ are the higher-order modes. The results shown were obtained by launching at $z = 0$ a Gaussian beam of width 8 $\mu$m and centered off-axis at $x = 4$ $\mu$m. The length of the waveguide was set at $L = 2$ cm.
Fig. 7. Modes and index profile (a), and mode spectrum (b) of a 1-D waveguide consisting of a low-index core sandwiched between two high-index layers. The core size, the thickness of each high-index layer and the index contrast between the core and the cladding were identical to those of the waveguide shown in Fig. 6 with ten high-index/low-index layer pairs in each Bragg mirror on either side of the core. The modes are labeled as m1, m2..., where m1 is the fundamental mode and m2,... are the higher-order modes. The launch condition and the length of the waveguide were the same as in Fig. 6.

5.3 Simple physical model

The above transmission spectra and mode-field patterns can be qualitatively understood by considering a simple model based on the diffraction of the Gaussian beam and the reflectivity at the core/high-index layer boundary. Consider the propagation of the beam launched symmetrically at $z = 0$ along the low-index core that is sandwiched between the high-index layers, as shown in Fig. 8 (a) below.

Fig. 8. (a) Simple model for deriving the reflection coefficient at the core/high-index layer boundary. The index of the core is $n_{\text{low}}$ and the index of each cladding layer is $n_{\text{high}}$. $E_i$, $E_r$, and $E_t$ are the incident, reflected and transmitted electric fields, respectively, at the boundary. $\theta$ is the half-angle of the diffracting Gaussian beam; $\theta_{\text{low}}$ and $\theta_{\text{high}}$ are the angles of incidence and refraction, respectively, at the interface. (b) Variation of the magnitude of the reflection coefficient for TE and TM polarized beams as a function of the spot size ($2w_0$) of the Gaussian beam and free-space wavelength ($\lambda_0$).
The propagating Gaussian beam diffracts with a half-angle given by \( \tan \theta = \frac{\lambda_0}{\pi w_0 n_{\text{low}}} \), where \( 2w_0 \) is the spot size of the beam [12]. The angle of incidence and the angle of refraction at the core/high-index layer boundary are \( \theta_{\text{low}} = (\pi/2) - \theta \) and \( \sin \theta_{\text{high}} = (n_{\text{low}}/n_{\text{high}}) \sin \theta_{\text{low}} \), respectively. Rough estimations of the reflection coefficient at the core/high-index layer boundary can be obtained by using Fresnel equations for TE as well as TM polarized beams. The absolute values of the reflection coefficients, \(|r|\), are shown in Fig. 8 (b) as a function of the free-space wavelength for different values of the spot size. For a given spot size and the wavelength range shown, the magnitude of the reflection coefficient for both TE and TM polarized beams decreases as wavelength increases, and it approaches unity in the limit of very short wavelengths. This qualitatively explains the tight confinement of the fundamental as well as the higher-order modes in the short-wavelength regime where light bounces off the core/high-index layer boundary at grazing incidence. The relative independence of the transmission spectrum to changes in the lattice constant in the short-wavelength regime is also qualitatively understood by the above model since most of the light is reflected off the first few low-index/high-index interfaces on either side of the core. As wavelength increases, the reflectivity at the boundary decreases so that more light can now leak into the outer layers of the Bragg mirrors, as confirmed by the mode shapes shown in Fig. 6 (a) and the drop in the core power observed in Fig. 3 (a) within the bandgap around \( \lambda_0 \approx 10 \mu \text{m} \). Coherent interference of waves reflected from the outermost layers of the Bragg reflectors also contributes increasingly to the core power within the bandgap as wavelength increases. Although polarization of the beam was taken into account in the above calculations, qualitative predictions of the simple model are still valid in interpreting the simulation results based on the scalar BPM as both the TE and TM polarizations show the same trend within the wavelength range shown in Fig. 8 (b).

The above analysis suggests that changing the width of the Gaussian beam should affect the transmission spectrum. This is indeed found to be the case when the beam width was varied in our simulations; the effect of decreasing the width from 8 \( \mu \text{m} \) to 4 \( \mu \text{m} \) is shown in Fig. 9 (a). With the exception of the varying beam width, the launch conditions were identical for the two sets of simulations, and the waveguide length was 5 cm in each case. The locations of the transmission minima and the bandgaps remain unchanged, but the transmission within the bandgap is reduced as the beam width decreases. Again, this can be understood by the increased diffraction of the launched Gaussian beam with a smaller spot size so that \( \theta_{\text{low}} \) and \( \theta_{\text{high}} \) are decreased, leading to a reduction in the reflectivity at the core/high-index layer boundary. Consequently, a fraction of the launched power can leak into the outer layers of the Bragg mirrors so that the power in the core decreases as the spot size of the Gaussian beam is decreased.

The above interpretation of the simulation results shown in Fig. 9 (a) is further supported by a modal analysis of waveguide W1. Computed mode shapes and spectra are shown in Figs. 9 (b) and (c), respectively, for an initial Gaussian beam of width 4 \( \mu \text{m} \) and launched off-axis at \( x = 4 \mu \text{m} \) at an exciting wavelength of 0.632 \( \mu \text{m} \). The propagation length was set to \( L = 2 \) cm for this calculation. The modes shown in Fig. 9 (b) should be compared to the modes shown in Fig. 6 (c) for a beam width of 8 \( \mu \text{m} \) with otherwise identical launch conditions. The mode spectrum obtained with the original beam width of 8 \( \mu \text{m} \) is also included in Fig. 9 (c) for comparison. The narrower Gaussian beam excites several higher-order modes. Consequently, the relative intensity of the fundamental mode is lower for a beam with a narrower width [see mode spectra in Fig. 9 (c)]. The highest-order mode (labeled m7) for the given launch condition also has a relatively larger optical field distribution within the outer layers of the Bragg mirrors compared to the lower-order modes [see Fig. 9 (b)]. In terms of the simple model discussed above, the narrower beam has a larger diffraction angle, and hence a larger spread of wavevectors in the transverse direction so that higher-order modes are excited.
Fig. 9 (a) Effect of varying the width of the launched Gaussian beam (centered at x = 0 at the input) on the transmission spectrum of the 1-D PBG waveguide W₁ of length L = 5 cm. (b) Modes excited at λ₀ = 0.632 µm in waveguide W₁ by launching a Gaussian beam of width 4 µm and centered off-axis at x = 4 µm. The length of the waveguide was set at L = 2 cm. (c) Comparison between the mode spectra for beam widths of 8 µm and 4 µm with otherwise identical launch conditions.
5.4 Propagation loss in PBG waveguides

The 1-D model was used to address the loss issue in PBG waveguides. The propagation loss is influenced by a number of factors, including the number and index contrast of the high-index/low-index layer pairs around the core as well as the absorption coefficients of the materials of the Bragg mirror and the core. Losses are due to both material absorption and waveguiding; the latter provides a minimum theoretical limit. In general, the length of the waveguide and the initial launch condition will also determine the transmission loss in the multi-moded waveguides studied in this paper since different modes will have different losses along the length of the waveguide, and the power in each mode will depend on the launch condition [see Fig. 9 (c) above]. Propagation loss over a specified length L of the waveguide and for a given launch condition is defined as \( \alpha_p = -10 \log_{10}(P_{\text{out}}/P_{\text{in}}) \) in units of dB, where \( P_{\text{in}} \) and \( P_{\text{out}} \) are the power launched into the core at \( z = 0 \) and the power transmitted out of the core at \( z = L \), respectively.

We calculated \( \alpha_p \) by propagating a Gaussian beam of width 8 \( \mu \)m centered at \( x = 0 \) at the input end of the low-index core over a distance \( L = 50 \) cm. In order to separate the material loss from the waveguide loss, the absorption loss of the high-index material (2.9 dB/cm) was included in one simulation while it was artificially set to zero in another one. The predicted total loss within the bandgap is about 0.022 dB at \( \lambda_0 = 0.612 \) \( \mu \)m over the waveguide length of 50 cm for computations in the wavelength range from 0.6-1 \( \mu \)m. The waveguide loss over the same length and the same wavelength range is about 0.0005 dB for the physical and material parameters chosen in our simulations. Clearly, material choice is critical in reducing propagation loss. Moreover, higher index contrasts between the low- and high-index regions would reduce the waveguide loss even further, as shown recently [14] for OmniGuide fibers.

6. Conclusions

A numerical model utilizing a scalar beam propagation method has been applied to understand how light propagates in PBG waveguides. It is shown that by reducing the cylindrical waveguide geometry to a planar waveguide model, important spectral features such as the locations of the transmission minima and maxima can be predicted numerically. These spectral features can be further calculated using an analytical ARROW model that is found to be in excellent agreement with the numerical results. Moreover, two different propagation regimes have been shown to exist depending on the wavelength. Bragg scattering off the outer high-index/low-index layer pairs becomes increasingly important for longer wavelengths while in the short-wavelength regime, the central low-index core and the first adjacent high-index layers of the waveguide predominantly determine its spectral characteristics. Mode computations and a simple physical model of reflection from the core/high-index layer boundary provide useful insight into the computed transmission spectra for the long- and the short-wavelength regimes.